**Geometry Mathematics Standards** 

# ARKANSAS MATHEMATICS STANDARDS

Algebra II & III Technical Math Quantitative Reasoning Advance Topics & Modeling Statistics Pre-Calculus & Calculus Critical Algebra I

Arkansas Division of Elementary & Secondary Education

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# DRAFT

# ALGEBRA II MATH STANDARDS

### **Radicals and Complex Numbers**

### **Radical Expressions and Rational Exponents**

Students apply properties to radical expressions and rational exponents.

- A2.RC.1: Explain how extending the properties of integer exponents to rational exponents provides an alternative notation for radicals.
- A2.RC.2: Apply the properties of exponents to translate between radical and exponential forms of expressions.
- A2.RC.3: Simplify and perform operations with radical expressions with and without variables; rationalizing denominators should include conjugates.

### **Complex Numbers**

Students apply properties to complex numbers.

- A2.RC.4: Know there is a complex number *i* and describe a context from which complex numbers appear.
- A2.RC.5: Perform the operations of addition, subtraction, multiplication, and conjugation of complex numbers.

A2.RC.6: Use polynomial identities with complex numbers.

### **Functions**

### Compositions

Students compose and compare functions.

A2.FN.1: Combine functions by addition, subtraction, multiplication, division, and composition to model the relationship between two quantities in mathematical and real-world contexts.

### Transformations

Students perform transformations in the coordinate plane.

- A2.FN.2: Decide if a function is even or odd from a graph or an algebraic expression.
- A2.FN.3: Explain how restricting the domain of a function allows the creation of its inverse.
- A2.FN.4: Write and graph the inverse of a given function; understand that the graph of an inverse function is a reflection of the function over the line y = x.

### Linear Functions, Equations, and Inequalities

### **Arithmetic Sequences**

Students use arithmetic sequences to model problems.

A2.LFE.1: Write and use arithmetic sequences recursively and explicitly to model situations; translate between the two forms when given a graph, a description of the relationship, or two input-output pairs.

**Algebra II Mathematics Standards** 

### Matrices

Students perform operations and matrices.

A2.LFE.2: Multiply a matrix by a scalar.

A2.LFE.3: Add and subtract matrices.

### Systems of Equations

Students solve systems of equations and inequalities.

A2.LFE.4: Solve systems of linear equations in three variables using matrices; use Gaussian elimination or technology.

### **Quadratic Functions & Equations**

### Create & Solve

Students create and solve problems that model quadratic relationships.

A2.QFE.1:	Calculate and inter	pret the averag	ge rate of change	of a qu	adratic functi	on represented	l in a table, g	raph, or as
	an equation in the	context of mat	hematical and rea	al-world	d problems.			

A2.QFE.2: Solve quadratic equations with complex number solutions.

A2.QFE.3: Represent and solve real-world problems using quadratic inequalities.

### Logarithms

Students graph and interpret key features of equations that model quadratic relationships.

### A2.QFE.4: Use the discriminant to determine the number and type of solutions of a quadratic equation.

A2.QFE.5: Sketch the graph of a quadratic function given a verbal description and show key features.

### Graph & Key Features

Students solve systems of equations.

A2.QFE.6: Solve a system of equations consisting of a *linear equation* and a nonlinear *equation* in two variables by choosing substitution, elimination, or graphically (with or without technology) as appropriate for the system of equations.

### **Exponential & Logarithmic Functions & Equations**

### Create & Solve

Students create and solve problems that model exponential and logarithmic relationships.

A2.ELF.1: Use the properties of exponents to find equivalent expressions and to solve equations, including those involving rational exponents.

- A2.ELF.2: Write and solve equations from real-world problems that can be represented as a logarithmic or exponential function in one variable.
- A2.ELF.3: Write exponential equations that model the relationship between two quantities when given a graph, a written description, or a table of values within a mathematical or real-world context.
- A2.ELF.4: Write and use geometric sequences recursively and explicitly to model situations; translate between the two forms when given a graph, a description of the relationship, or two input-output pairs.

### Logarithms

Students define and use logarithms.

A2.ELF.5:	Translate between logarithmic and exponential forms of an equation.
A2.ELF.6:	Use properties of logarithms to simplify and evaluate logarithmic expressions, with or without technology.
A2.ELF.7:	Use the inverse relationship between exponents and logarithms to solve problems.

### **Graph & Key Features**

Students graph and interpret key features of exponential and logarithmic models.

A2.ELF.8:	Determine the domain and range of logarithmic functions in mathematical problems.
A2.ELF.9:	Determine reasonable domain and range values of logarithmic functions representing real-world situations,
	both continuous and discrete; interpret the solution as reasonable or unreasonable in context.
A2.ELF.10:	Sketch the graph of an exponential function given a verbal description and show key features.
A2.ELF.11:	Calculate and interpret the average rate of change of an exponential function represented in a table, graph, or
	as an equation in the context of mathematical and real-world problems.
A2.ELF.12:	Graph exponential and logarithmic functions with and without context, identifying key features, and
	determining constraints in a given context.
A2.ELF.13:	Graph and generalize the effect of transformations on exponential and logarithmic functions.
	Transformations include: stretches, compressions, vertical shifts, and horizontal shifts
A2.ELF.14:	Given the graphs of exponential and logarithmic functions, explain the effects of the transformation from the

- A2.ELF.14: Given the graphs of exponential and logarithmic functions, explain the effects of the transformation from the parent function.
  - Exponentials:  $y = ab^x$ ,  $a \neq 0$ , b > 0, and  $b \neq 1$
  - Logarithms:  $y = log_b(x), b > 0, x > 0$  and  $b \neq 1$

### Polynomial, Rational, & Other Functions & Equations

### Create & Solve

Students create and solve rational and polynomial equations.

- A2.PRF.1: Write and solve equations from real-world problems that can be represented as a rational or square root function in one variable.
- A2.PRF.2: Solve non-linear formulas for a specified variable.
- A2.PRF.3: Find zeros of polynomial functions with a degree of 3 or higher when suitable factorizations are available in a real-world and mathematical context.

- A2.PRF.4: Use zeros and an understanding of multiplicity to sketch a graph of a polynomial function with a degree of 3 or higher.
- A2.PRF.5: Apply the Fundamental Theorem of Algebra to determine the number and potential types of roots of polynomial functions based on the degree of the polynomial.
- A2.PRF.6: Solve rational and radical equations containing one variable specifying extraneous solutions.

### **Graph & Key Features**

Students graph and identify key features of functions.

- A2.PRF.7: Determine the domain and range of polynomial and rational functions in mathematical problems.
- A2.PRF.8: Determine reasonable domain and range values of polynomial and rational functions representing real-world situations, both continuous and discrete; interpret the solution as reasonable or unreasonable in context.
- A2.PRF.9: Interpret the key features of polynomial functions that model a relationship between two quantities in a given context; translate between different representations of the function, especially graphs, tables, and equations.
- **A2.PRF.10:** Sketch the graph of a polynomial function given a verbal description and show key features.
- A2.PRF.11: Calculate and interpret the average rate of change of polynomial functions represented in a table, graph, or as an equation in context of mathematical and real-world problems.
- A2.PRF.12: Graph functions with and without context, identifying key features and determining constraints in a given context.
  - Polynomial functions
  - Rational functions
  - Square root functions
  - Piecewise-defined functions
- A2.PRF.13: Graph and generalize the effect of transformations on square root, cubic, and rational functions.
  - Transformations include: stretches, compressions, vertical shifts, and horizontal shifts
- A2.PRF.14: Given a graph, explain the effects of the transformation from the parent function.
  - Functions include: square root  $y = \sqrt{x}$ , cubic  $y = x^3$  and rational functions  $y = \frac{1}{x}$

### **Factor Polynomials**

Students factor polynomials.

**A2.PRF.15:** Apply the Remainder Theorem to factor polynomial functions.

A2.PRF.16: Verify polynomial identities and use them to describe numerical relationships.

### **Rational Expressions**

Students perform operations on rational expressions.

- A2.PRF.17: Apply understanding of rational number operations to add, subtract, multiply, and divide by nonzero rational expressions.
- A2.PRF.18: Rewrite simple rational expressions in different forms.
- A2.PRF.19: Divide polynomial expressions using inspection, long division, and synthetic division, with and without a remainder.

### Systems of Equations

Students solve systems of equations.

- A2.PRF.20: Explain why a solution to the equation f(x) = g(x) is the x-coordinate where the y-coordinate of f(x) and g(x) are the same using graphs, tables, or approximations.
  - Include cases where f(x) and/or g(x) are linear, polynomial, exponential, logarithmic, or rational and where at least one of the functions is not linear.

### **Statistics & Probability**

### **Statistical Experiments & Studies**

Students evaluate processes for statistical experiments, make inferences, and justify conclusions from statistical studies.

- A2.SP.1: Use data from a random sample to make inferences about a population.
- A2.SP.2: Compare theoretical and empirical probabilities using simulations.
- A2.SP.3: Distinguish between sample surveys, experiments, and observational studies and explain the purpose of randomization in statistical studies.
- A2.SP.4: Read and explain, in context, the validity of data from outside reports by:
  - Identifying the variables as quantitative or categorical.
  - Describing how the data was collected.
  - Indicating any potential biases or flaws.
  - Identifying inferences the author of the report made from sample data.

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# **TECHNICAL MATH STANDARDS**

### Mathematical Processes & Modeling

### Mathematical Processes & Modeling

Students use mathematical processes and models to acquire, demonstrate, and communicate mathematical understanding in real-world scenarios.

- **TM.MPM.1**: Apply mathematics to problems arising in everyday life, workplace, and society.
- **TM.MPM.2:** Use mathematical processes with algebraic formulas, numerical techniques, and graphs to solve real-world scenarios.
- **TM.MPM.3:** Create mathematical models and use problem-solving skills, independently and as a collaborative team, for real-world scenarios to:
  - Analyze given information or data
  - Identify patterns or relationships
  - Formulate a plan or strategy
  - Estimate solutions
  - Determine a solution
  - Justify a solution and its reasonableness
  - Describe limitations
  - Identify how results are affected by changing parameters
  - Suggest improvements
- **TM.MPM.4:** Select appropriate tools and techniques to solve problems.

TM.MPM.5: Demonstrate effective use of resources.

**TM.MPM.6:** Use precise mathematical language and multiple representations to organize, record, and communicate mathematical ideas or solutions to solve real-world scenarios independently and collaboratively.

### Numerical & Proportional Reasoning

### Model with Estimation

Students use estimation to solve real-world problems and assess the reasonableness of a solution.

**TM.NPR.1:** Use estimation to identify the most reasonable mathematical solution.

**TM.NPR.2:** Use estimation and precision in real-world scenarios.

### **Proportional Reasoning**

Students understand and reason about relationships between quantities.

- **TM.NPR.3:** Solve real world-problems and interpret results involving calculations with percentages, decimals, and fractions.
  - Conversions
  - Percent change

- Percent of quantities
- **TM.NPR.4:** Recognize, set up, and solve proportions from real-world scenarios.
- **TM.NPR.5:** Utilize real-world scenarios requiring interpretation and comparison of various representations of rates, ratios, and proportions including scale drawings.

### Comparison

Students use number sense and proportional reasoning to draw conclusions and communicate results.

- **TM.NPR.6:** Compare magnitudes of numbers in context in different forms.
- TM.NPR.7: Use dimensional analysis to solve problems involving multiple units of measurement.

### Algebraic Relationships

### **Model with Functions**

Students use mathematical concepts of algebra to explain linear and nonlinear applications in real-world scenarios.

- TM.AR.1: Analyze and apply rate of change in terms of real-world scenarios.
- TM.AR.2: Use concepts of systems of equations and inequalities to model and solve real-world scenarios.
- TM.AR.3: Use linear programming with or without the use of technology to:
  - Maximize or minimize (optimize) linear objective function in real-world scenarios.
  - Determine the reasonableness of solutions.

### Model with Data

Students use data to make decisions and predictions.

- **TM.AR.4:** Collect and organize data, independently and as a collaborative team, to create appropriate graphical representations of real-world scenarios.
  - Interpret graphical representations.
  - Make predictions and decisions based on representations.
  - Analyze results based on representations.
- TM.AR.5: Create, interpret, and analyze best-fit models of linear and exponential functions to solve real-world scenarios.
  - Interpret the constants, coefficients, and bases in the context of the data.
  - Check the model for best fit and use the model, where appropriate, to draw conclusions or make predictions.

### Measurement

### Measurement & Measurement Tools

Students apply measurement and use measurement tools in real-world scenarios.

TM.MS.1: Convert between and within the metric system and the U.S. customary system in real-world scenarios.

- TM.MS.2: Demonstrate mastery of utilizing measuring devices:
  - Apply accurate readings of both metric and the U.S. customary measuring devices to a problem situation.
  - Select and use appropriate measuring devices and understand the limitations of such devices for realworld scenarios.
- **TM.MS.3:** Determine and use appropriate unit labels for real-world scenarios.
  - Unit labels include: length, weight, capacity, distance, temperature, time, surface area, volume, area, perimeter.

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# DRAFT

# QUANTITATIVE REASONING STANDARDS

### Modeling

### Problem Solving

Students use appropriate mathematical models to solve problems involving everyday life, workplace, and society.

**QR.MD.1:** Use mathematical models to:

- Demonstrate understanding of the meaning of a solution in context.
- Identify when insufficient information is given to solve a problem.
- QR.MD.2: Analyze mathematical models, describe limitations, and suggest improvements.
- **QR.MD.3:** Use mathematical models created with spreadsheets or other tools to:
  - Estimate solutions for contextual questions.
  - Identify patterns.
  - Identify how changing parameters affect results.
- **QR.MD.4:** Use mathematical models to make decisions about purchases.

### **Bivariate Data Sets**

Students use bivariate data sets to solve problems.

- **QR.MD.5:** Select models for a given set of bivariate data sets; justify the choice.
- QR.MD.6: Represent and use mathematical models for bivariate data sets to answer questions, draw conclusions, and make decisions.

### Numerical Reasoning

### Number, Ratio, & Probability

Students use number sense and proportional reasoning in real-world settings to make and communicate decisions in order to draw conclusions based on quantitative analysis.

QR.NR.1:	Solv	Solve real world problems and interpret results involving calculations with percentages, decimals, and		
	frac	tions.		
	•	Problem types include: conversions, percent change (absolute vs relative), and percent of quantities.		

- **QR.NR.2:** Use estimation in real world situations.
- **QR.NR.3:** Numeric and contextual benchmarks:
  - Identify appropriate numeric benchmarks for estimating calculations.
  - Identify appropriate contextual benchmarks to compare to other numbers.
- QR.NR.4: Compare magnitudes of numbers in context in different forms.
- QR.NR.5: Use dimensional analysis to solve problems involving multiple units of measurement.
- **QR.NR.6:** Solve real-world problems requiring interpretation and comparison of various representations of rates and ratios.

**QR.NR.7:** Distinguish between proportional and non-proportional real-world situations; when appropriate, apply proportional reasoning.

### **Statistics & Probability**

### Statistical

Students will draw conclusions, make decisions, and communicate based on understanding using statistical information.

- **QR.SP.1:** Create charts, tables, and graphs of real world data with and without technology.
- QR.SP.2: Analyze and interpret charts, tables, and graphs using real world data.
- QR.SP.3: Compare and contrast charts, tables, and graphs using real world data.
- **QR.SP.4:** Analyze statistical information from studies, surveys, and polls to make informed judgements as to the validity of claims or conclusions.
- **QR.SP.5:** Make decisions about data summarized numerically using measures of center:
  - Compare measures of center of two or more data sets.
  - Interpret the differences in context.
  - Justify the use of a chosen measure.

### **Probabilities**

Students apply probabilistic reasoning to draw conclusions, to make decisions, and to evaluate outcomes of decisions.

- **QR.SP.6:** Use probabilities to make and justify decisions about risks in everyday life.
- **QR.SP.7:** Evaluate the validity of claims based on experimental and theoretical probabilities.

QR.SP.8: Apply rules of counting and probability to compute probabilities of compound real world events:

- Addition Rule of Probability
- Multiplication Rule of Probability
- Fundamental Counting Principle
- Permutation and combinations
- Visual representations

### Personal Financial Literacy

### Employment/Income

Students apply mathematics to make informed employment and income decisions.

- **QR.PF.1:** Represent and analyze mathematical models for various types of income.
- **QR.PF.2:** Represent and analyze various types of income deductions and employment forms.
- **QR.PF.3:** Analyze expenses to create a household budget utilizing food, shelter, transportation, utilities, insurance, savings, and other expenses.

### Investment

Students apply mathematics to make informed investment decisions.

- **QR.PF.4:** Analyze various investment instruments for:
  - Purposes
  - Advantages
  - Disadvantages
  - Risks

### Credit & Debt

Students apply mathematics to make informed credit and debt decisions.

- **QR.PF.5:** Analyze the characteristics of various types of loans.
- **QR.PF.6:** Apply appropriate models to determine the impact of the relationship among loan rates, the term of a loan, the principal amount of a loan, and payments.
- **QR.PF.7:** Examine consumer protection, bankruptcy, and credit and debt management services for ways in which they affect household budgeting.

### **Business Financial Literacy**

### **Business & Economics**

Students understand the principles and mathematics in business as it applies to economics.

- QR.BF.1: Use real-world data to determine how a product or service can be profitable in a community.
- QR.BF.2: Determine fixed and variable expenses of running a business.
- **QR.BF.3:** Calculate indices and solve problems using common indices:
  - Consumer price index
  - Cost of living index
  - Determine what constitutes an index
- QR.BF.4: Analyze how stock market averages and indices are calculated with technology.
- **QR.BF.5:** Research how inflation changes the value of the U.S. Dollar over time.
- **QR.BF.6:** Prepare for employment by analyzing job skills.

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# ADVANCED TOPICS & MODELING IN MATHEMATICS STANDARDS

### **Functions**

### Interpreting Functions

Students extend previous knowledge of functions beyond linear and quadratic.

- AT.FN.1: Interpret key features of graphs and tables in terms of two quantities, which extend to function families beyond linear and quadratic, that model a relationship between the quantities in a contextual application and/or student-generated data.
- AT.FN.2: Analyze mathematical models, describe limitations, and suggest improvements.

### **Graphing Functions**

Students analyze function using graphing.

- AT.FN.3: Graph functions expressed symbolically and show key features of the graph using technology.
  - Functions include exponential, logarithmic, and trigonometric functions.
- AT.FN.4: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions, with or without the appropriate technology.
  - Contextual situations may include: cube root (e.g., minimizing packaging on cubic boxes, geostationary satellites), piecewise (e.g., postage stamp function, teacher salary, GPS for distance), and square root (distance via Pythagorean Theorem)
- AT.FN.5: Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior, with or without the appropriate technology.
- AT.FN.6: Graph rational functions, identifying zeros and asymptotes (vertical, horizontal, and/or oblique) when suitable factorizations are available and showing end behavior, with or without the appropriate technology.
- AT.FN.7: Graph exponential and logarithmic functions, showing intercepts and end behavior.
- AT.FN.8: Graph trigonometric functions showing period, midline, and amplitude.

### Vectors & Matrices

### Vectors

Students represent and model vector quantities and perform operations on vectors.

- **AT.VM.1:** Recognize that vector quantities have both magnitude and direction and can be represented by directed line segments.
- AT.VM.2: Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- AT.VM.3: Solve problems involving velocity and other quantities that can be represented by vectors.
- AT.VM.4: Add and subtract vectors graphically and algebraically.
- AT.VM.5: Given two vectors in magnitude and direction form, determine the magnitude and direction of the sum.
- AT.VM.6: Multiply a vector by a scalar graphically and analytically; reverse their direction when possible.

**AT.VM.7:** Compute the magnitude and direction of a vector by multiplying a vector by a scalar.

### Matrices

Students perform operations on matrices and use matrices in applications.

- AT.VM.8: Use matrices to represent, list, describe and manipulate data with technology.
- **AT.VM.9:** Multiply a matrix by a scalar.
- AT.VM.10: Add and subtract matrices.
- AT.VM.11: Multiply matrices, understanding that matrix multiplication for square matrices is not commutative.
- **AT.VM.12**: Represent a system of linear equations as a single matrix equation in a vector variable.
- **AT.VM.13:** Find and use the inverse of a matrix to solve systems of linear equations; solve  $3 \times 3$  or greater systems of equations with technology.

### Statistics & Probability

### **Expected Value**

Students calculate and use expected values to solve problems.

AT.SP.1:	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample
	space; graph the corresponding probability distribution using the same graphical displays as for data
	distributions.
AT.SP.2:	Calculate the expected value for a random variable and describe the expected value as the mean of the
	probability distribution in context.
AT.SP.3:	Create a probability distribution using theoretical probabilities; calculate the expected value.
AT.SP.4:	Create a probability distribution using experimental or observational data; calculate the expected value.

### **Decisions Using Probability**

Students use probability to evaluate outcomes of decisions.

- AT.SP.5: Analyze the costs and benefits of possible outcomes of making a decision by assigning probabilities to particular payoff values; calculate expected values.
- AT.SP.7: Analyze decisions and strategies using probability concepts.

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# CALCULUS STANDARDS

### Limits & Continuity

### Identify and Calculate Limits

Students will determine the limit of a function at a value numerically, graphically, and analytically.

- **C.LC.1:** Identify vertical asymptotes in rational and logarithmic functions by identifying locations where the function value approaches infinity; estimate limits numerically and graphically; calculate limits analytically:
  - Algebraic simplification
  - Direct substitution
  - One-sided limits
  - Rationalization
- **C.LC.2:** Calculate infinite limits and use the result to identity vertical asymptotes in rational and logarithmic functions.
- **C.LC.3:** Calculate limits at infinity and use the result to identify horizontal asymptotes in rational and exponential functions.
- C.LC.4: Calculate limits at infinity and use the result to identify unbounded behavior in rational, exponential, and logarithmic functions.

**C.LC.5:** Identify and classify graphically, algebraically, and numerically if a discontinuity is removable or non-removable; identify the three conditions that must exist in order for a function to be continuous at x = a:

- f(a) is defined
- The limit as x approaches a of f(x) equals f(a)
- The limit as x approaches a of f(x) exists

C.LC.6: Apply the Intermediate Value Theorem for continuous functions.

### **Derivatives**

### Equation of a Tangent Line

Students will use derivatives to solve problems both theoretically and in real-world context.

### C.D.1: Approximate the derivative:

- Graphically by finding the slope of a tangent line drawn to a curve at a given point.
- Numerically by using the difference quotient.
- **C.D.2:** Find the equation of the tangent line using the definition of derivative.
- **C.D.3:** Establish and apply that differentiability implies continuity, but continuity does not necessarily imply differentiability.
- **C.D.4:** Compare the characteristic of graphs of *f* and *f*':
  - Generate the graph of *f* given the graph of *f*' and vice versa.
  - Establish the relationship between the increasing and decreasing behavior of *f* and the sign of *f'*.
  - Identify maxima and minima as points where increasing and decreasing behavior change.
- C.D.5: Apply the Mean Value Theorem on a given interval.

- **C.D.6:** Compare the characteristic of graphs of f, f', and f'':
  - Generate the graphs of *f* and *f*' given the graph of *f*" and vice versa.
  - Establish the relationship between the concavity of *f* and the sign of *f*".
  - Identify points of inflection as points where concavity changes.
- C.D.7: Find derivatives of functions using:
  - Power rule
  - Product rule
  - Quotient rule Find derivatives of:
- C.D.8:
- An implicitly defined equation
- Composite functions using chain rule
- Exponential and logarithmic functions
- Functions requiring the use of more than one differentiation rule
- C.D.9: Find the equation of:
  - A line tangent to the graph of a function at a point
  - A normal line to the graph of a function at a point
- C.D.10: Solve application problems involving:
  - Optimization
  - Related rates
- C.D.11: Interpret the derivative as a rate of change and varied applied contexts.
  - Contexts include: velocity, speed, and acceleration

### **Integrals**

### Define the Definite Integral

Students will apply techniques of integration to solve problems, both theoretically and in contextual models that represent real-world phenomena.

- **C.I.1:** Define the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval.
  - If f is a real, continuous function defined on [a, b] and F is an antiderivative of f in [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a).$$

- **C.I.2:** Determine the area between two curves and identify the definite integral as the area of the region bounded by two curves.
- **C.I.3:** Apply the Fundamental Theorem of Calculus to solve contextual models that represent real-world phenomena.
- C.I.4: Find the general solution to indefinite integrals.
- **C.I.5:** Determine the antiderivative of a function using rules of basic differentiation, and solve problems using the techniques of antidifferentiation including but not limited to power rule and *u*-substitution.
- **C.I.6:** Estimate definite integrals by using Riemann sums (left, right, midpoint, and trapezoidal) and identify the definite integral as a limit of Riemann sums.
- C.I.7: Explore applications of integration.

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# DRAFT

# ALGEBRA III STANDARDS

### **Complex Numbers and Conic Sections**

### **Complex Numbers**

Students apply properties to complex numbers.

A3.CNC.1: Find the conjugate of a complex number; use conjugates to find quotients of complex numbers.

### **Conic Sections**

Students relate the equations and graphs of conic sections.

- A3.CNC.2: Generate an equivalent form of an equation for a conic section by completing the square to identify key characteristics.
  - Conic sections include: circles, ellipses, parabolas, and hyperbolas
- A3.CNC.3: Calculate infinite limits and use the result to identity vertical asymptotes in rational and logarithmic functions.

### **Functions**

### Compositions

Students compose and compare functions.

**A3.FN.1:** Combine functions by addition, subtraction, multiplication, division, and composition to model the relationship between two quantities in real-world and mathematical contexts.

### Inverses

Students find inverse functions.

- A3.FN.2: Verify if two functions are inverses of each other using composition of functions.
- A3.FN.3: For a function with an inverse, explain how to read the ordered pairs of the inverse function when given a graph or table of values.
- A3.FN.4: Construct an invertible function from a non-invertible function by restricting the domain.

### Transformations

Students graph function transformations.

- A3.FN.5: Graph and generalize the effect of transformations on quadratic, absolute value, square root, cube root, cubic, and step functions.
  - Transformations include: stretches, compressions, vertical shifts, and horizontal shifts
- A3.FN.6: Determine if a function is even, odd, or neither from a graph or an algebraic expression.

### Sequences

Students use sequences to model and analyze mathematical situations.

- A3.FN.7: Write and use arithmetic and geometric sequences recursively and explicitly to model situations, translating between the two forms.
  - Forms include: when given a graph, a description of the relationship, or two input-output pairs •

### **Matrices**

### **Operations**

Students represent and perform operations with matrices.

- A3.MAT.1: Use matrices to describe, list, and manipulate data in different situations.
- A3.MAT.2: Multiply matrices by scalars to solve real-world and mathematical problems.
- A3.MAT.3: Add and subtract matrices.
- A3.MAT.4: Multiply matrices, understanding that matrix multiplication for square matrices is not commutative.
- A3.MAT.5: Calculate the determinant of a square matrix to determine if it has an inverse.

### Systems

Students use matrices to solve systems of equations.

A3.MAT.6: Solve systems of linear equations using augmented matrices.

### **Exponential and Logarithmic Functions**

### Analyze & Interpret

Students analyze and interpret exponential and logarithmic functions.

- Analyze and interpret exponential and logarithmic functions, identifying key characteristics. A3.ELF.1:
  - Functions should be represented numerically, graphically, and algebraically. •
  - Key features include asymptotes, end behavior, intercepts, domain, and range.

### Solve

Students solve problems with exponential and logarithmic functions.

A3.ELF.2: Understand and apply the inverse relationship between exponents and logarithms to solve problems.

### Polynomial & Rational Functions

### Analyze, Interpret, & Graph

Students analyze, interpret, and graph polynomial and rational functions.

A3.PRF.1: Analyze and interpret polynomial functions, identifying key characteristics.

- Functions should be represented numerically, graphically, and algebraically.
- Key features include end behavior, intercepts, domain, range, relative and absolute maximum and minimum, and intervals over which the function is increasing or decreasing.
- A3.PRF.2: Analyze and interpret rational functions, identifying key characteristics.
  - Functions should be represented numerically, graphically, and algebraically.
  - Key features include asymptotes (vertical, horizontal, and slant), end behavior, point discontinuities, intercepts, domain, and range.
- A3.PRF.3: Graph rational functions showing zeros, asymptotes, and end behavior.



## PRE-CALCULUS STANDARDS

### Vectors & Matrices

Recognize, Model, & Write Vectors

Students recognize, model, and write vector quantities.

- **PC.VM.1:** Recognize that vector quantities have both magnitude and direction and can be represented by directed line segments.
- PC.VM.2: Write vector quantities using appropriate symbols indicating magnitude and direction.

### **Vector Operations**

Students perform operations involving vectors.

- **PC.VM.3:** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- PC.VM.4: Solve problems involving velocity and other quantities that can be represented by vectors.
- PC.VM.5: Add and subtract vectors graphically and algebraically.
- PC.VM.6: Multiply a vector by a scalar graphically and analytically; reverse their direction when possible.

### **Matrix** Operations

Students represent and perform operations with matrices.

- **PC.VM.7:** Use matrices to list, describe, and manipulate data with and without technology.
- PC.VM.8: Multiply matrices, understanding that matrix multiplication for square matrices is not commutative.
- **PC.VM.9:** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
- **PC.VM.10:** Work with 2 × 2 matrices as transformations of the plane; interpret the absolute value of the determinant in terms of area

### Trigonometry

### Radians

Students understand, explain, and describe radian measure.

- **PC.TR.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- PC.TR.2: Convert between radian and degree measure.
- **PC.TR.3:** Explain how the unit circle can be used to model sine, cosine, tangent, secant, cosecant, and cotangent for all real numbers.

### Unit Circle

Students use the unit circle to express and find exact values for trigonometric functions.

- **PC.TR.4:** Construct special right triangles on the unit circle to find the exact values of sine, cosine, tangent for  $\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}, \text{ and } \frac{\pi}{2}$ .
- **PC.TR.5:** Use the unit circle to express the values of sine, cosine, and tangent for  $\pi x$ ,  $\pi + x$ , and  $2\pi x$  in terms of their exact values for x, where x is one of these values:  $\frac{\pi}{3}$ ,  $\frac{\pi}{4}$ ,  $\frac{\pi}{6}$ , and  $\frac{\pi}{2}$ .

### Identities, Formulas, & Laws

Students develop and apply identities, formulas, and laws using trigonometry.

- **PC.TR.6:** Develop the Pythagorean identity,  $sin^2(\theta) + cos^2(\theta) = 1$ .
- **PC.TR.7:** Apply the Pythagorean identity to find the remaining trigonometric functions when given  $sin(\theta)$ ,  $cos(\theta)$ , or  $tan(\theta)$  and the quadrant of the angle.
- **PC.TR.8:** Develop addition, subtraction, double, and half-angle formulas for sine, cosine, and tangent and use them to solve problems, including verifying other identities.
- **PC.TR.9:** Develop the formula for the area of a triangle,  $A = \left(\frac{1}{2}\right)ab \sin C$ , using trigonometry.
- PC.TR.10: Develop and apply the Law of Sines and the Law of Cosines to solve real-world and mathematical problems including finding unknown measurements in right and non-right triangles.
- **PC.TR.11:** Define and use reciprocal functions, cosecant, secant, and cotangent to solve problems.

### Solve & Graph

Students explore, solve, and sketch the graphs of periodic trigonometric functions.

- **PC.TR.12:** Explain whether a trigonometric function is even or odd and recognize the periodicity of the graph using the unit circle.
- PC.TR.13: Graph trigonometric and inverse trigonometric functions and show period, midline, and amplitude.
- PC.TR.14: Select a trigonometric function that models real-world contexts.
- PC.TR.15: Explain how restricting the domain of a trigonometric function allows the creation of its inverse.
- **PC.TR.16:** Solve and evaluate the solution of trigonometric equations in real-world contexts; interpret the solution in terms of its context.
- **PC.TR.17:** Recognize that some trigonometric equations have infinitely many solutions and be able to state a general formula to represent the infinite solutions.
- **PC.TR.18:** Calculate and interpret the average rate of change over a specified interval of a trigonometric function represented in a table, graph, or as an equation in the context of real-world and mathematical problems.

### Conic Sections

### Derive Equations

Students derive equations for conic sections.

- **PC.CS.1:** Derive the general form of the equation of a circle using the Distance Formula or Pythagorean Theorem.
- **PC.CS.2:** Derive the equation of a parabola given a focus and directrix.
- PC.CS.3: Derive the equations of ellipses and hyperbolas given the foci using the Distance Formula.

### **Explore Equations**

Students use matrices to solve systems of equations.

- PC.CS.4: Find the equations for the asymptotes of a hyperbola.
- **PC.CS.5:** Generate an equivalent form of an equation for a conic section by completing the square to identify key characteristics.
  - Conic sections include: circles, ellipses, parabolas, and hyperbolas
- **PC.CS.6:** Identify, graph, write, and analyze equations of each type of conic section using properties and technology when appropriate.
  - Conic sections include: circles, ellipses, parabolas, and hyperbolas
  - Properties include: symmetry, intercepts, foci, asymptotes, and eccentricity

### Systems of Equations & Inequalities

Students solve systems of equations and inequalities involving conic sections.

- PC.CS.7: Solve systems of equations and inequalities involving conics and other types of equations, with and without technology.
  - Equations include: conic-conic and conic-linear

### **Functions**

### Solve Problems

Students derive and apply functions.

- **PC.FN.1:** Understand that sequences are functions, sometimes defined recursively, whose domains are a subset of the integers.
- **PC.FN.2:** Derive the formula for the sum of a finite geometric series; apply the formula to solve conceptual problems.
- **PC.FN.3:** Apply the Binomial Theorem for the expansion of  $(a + b)^n$  in powers of a and b for a positive integer n, where a and b are any number.
- **PC.FN.4:** Build functions to model real-world contexts using algebraic operations on functions and composition, with and without appropriate technology.

### **Explore Graphing**

Students graph and interpret functions.

- **PC.FN.5:** Graph power and polynomial functions, identify zeros (when suitable factorizations are available), and show end behavior.
- **PC.FN.6:** Graph rational functions, identify zeros and asymptotes (when suitable factorizations are available), and show end behavior.
  - Asymptotes include: horizontal, vertical, and oblique

- **PC.FN.7:** Graph exponential and logarithmic functions; show intercepts and end behavior.
- **PC.FN.8:** Compare key features of two functions each represented in a different way.
  - Representations include: algebraic, graphic, numeric in tables, and verbal descriptions

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# DRAFT

## STATISTICS STANDARDS

### Making Inferences & Justifying Conclusion

### Surveys, Experiments, & Observational Data

Students make inferences and justify conclusions from sample surveys, experiments, and observational studies.

- S.MI.1: Estimate a population mean or proportion from a sample survey; develop a margin of error through the use of simulation models for random sampling.
- Calculate the standardized test statistic and P-value for a test about a population proportion and a population S.MI.2: mean; determine if the sample data provides convincing evidence against a parameter claim.
- S.MI.3: Compare two treatment groups in an experiment and determine if the difference in parameters is significant by calculating the standardized test statistics and P-value.

### **Conditional Probability & Rules of Probability**

### **Compute Probability of Compound Events**

Students use the rules of probability to compute probabilities of compound events.

Determine unions or intersections of events in a sample space; determine complements of events. S.RP.1: S.RP.2: Identify the two components that make up a legitimate probability model/distribution.

### Independence & Conditional Probability

Students understand and use independence and conditional probability to interpret data.

- S.RP.3: Determine if two events, A and B, are independent when given the probabilities of A and B.
- S.RP.4: Calculate and use conditional probabilities to determine if events are independent.
- Create and analyze two-way frequency tables of data to calculate marginal, joint, and conditional probabilities. S.RP.5:
- S.RP.6: Using a two-way table, determine if two events are independent.
- S.RP.7: Explain conditional probability and independence using everyday language in a variety of real-world contexts.
- S.RP.8: Find the conditional probability of A given B, P(A|B), and interpret the answer in terms of the model, including two-way frequency tables and Venn diagrams.
- Apply the Addition Rule, P(A or B) = P(A) + P(B) P(A and B) and interpret the answer. S.RP.9:
- S.RP.10: Identify whether or not two events are mutually exclusive / disjoint.
- Apply the general Multiplication Rule, P(A and B) = P(A)P(B|A) = P(B)P(A|B) and interpret the answer. S.RP.11:
- S.RP.12: Compute the probability of compound events and solve problems using combinations, permutations, Venn Diagrams, and Tree Diagrams.

### Using Probability to Make Decisions

### **Expected Values**

Students calculate and use expected values of random variables to solve problems.

- **S.PMD.1:** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
- **S.PMD.2:** Calculate the expected value for a discrete random variable; describe the expected value as the mean or typical value of the probability distribution in context.
- **S.PMD.3:** Create a probability distribution of a discrete random variable using theoretical probabilities and use the probability distribution to calculate the probability of an event.
- **S.PMD.4:** Create a probability distribution for a discrete random variable using experimental or observational data; calculate the expected value.
- **S.PMD.5:** Use probability density models such as the normal curve and uniform density curve to model real-world data; calculate probabilities of continuous random variables using these models.

### **Outcomes of Decision**

Students evaluate outcomes of decisions using probability.

**S.PMD.6:** Analyze the costs and benefits of possible outcomes of making a decision by assigning probabilities to particular payoff values of a discrete random variable and calculate expected values.

### Collecting Data & Data Bias

### **Data Collection**

Students explore best practices of collecting data while identifying possible sources of bias in data collection methods.

- S.CD.1: Describe how to take a simple random sample using technology or a random number table.
- **S.CD.2:** Use randomization strategies to ensure random selection processes are fair.
- **S.CD.3:** Understand that certain types of sampling methods may lead to bias, such as convenience and voluntary samples.

### **Displaying & Describing Distributions of Data**

### **Data Representation**

Students represent raw data in tabular and graphical form to describe features of the data and summarize trends.

- **S.DD.1:** Distinguish between categorical and quantitative data.
- **S.DD.2:** Determine if there is an association between two quantitative variables using the correlation coefficient and scatter plots.
- **S.DD.3:** Model real-world data using least squares regression techniques.

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# DRAFT

# **CRITICAL ALGEBRA I MATH STANDARDS**

*Critical Algebra I Math Standards are composed of the Algebra I standards and supporting standards from previous grades. The supporting standards are located at the end of this course.* 

### **Expressions**

### Polynomials, Roots, & Exponent Laws

Students simplify algebraic and numerical expressions.

- A1.EX.1: Add, subtract, and multiply polynomials; compare the system of polynomials to the system of integers when performing operations.
  - Supporting standards: 6.ALG.5, 7.ALG.1, 8.NCC.3
- A1.EX.2: Simplify and perform operations with radical expressions without variables; rationalizing denominators should not include conjugates.
  - Supporting standards: 6.NCC.11, 8.NCC.8
- A1.EX.3: Simplify algebraic expressions using the laws of exponents.
  - Supporting standards: 6.ALG.3, 7.NCC.7, 7.NCC.8, 8.NCC.3

### A1.EX.4: Interpret the parts of expressions such as terms, factors, and coefficients in terms of a real-world context.

• Supporting standards: 5.CAR.13, 6.ALG.1, 6.ALG.2

### Functions

### Domain & Range, Function Notation

Students understand the concept of a function, domain and range, and use function notation; students use function notation to solve problems.

- A1.FN.1: Explain that a function assigns each element in the domain to exactly one element in the range.
  - Supporting standards: 7.ALG.5, 8.FN.3
- A1.FN.2: Use function notation to represent functions, understanding that if f is a function and x is an element of its domain, then f(x) represents the output of f corresponding to the input x.
  - Supporting standards: 7.ALG.5, 8.FN.3
- A1.FN.3: Graph functions given in function notation, understanding that the graph contains the points (x, f(x)).
  - Supporting standards: 5.CAR.12, 6.ALG.3, 6.ALG.6, 7.NCC.9
- A1.FN.4: Evaluate functions expressed in function notation for one or more elements in their domains (inputs); use function notation to describe a contextual situation.
  - Supporting standards: 7.ALG.5, 8.FN.3

### Construct & Compare

Students construct and compare linear, quadratic, and exponential models and solve problems.

- A1.FN.5: Differentiate between real-world scenarios that can be modeled by exponential or linear functions by determining whether the relationship has a common difference or a common ratio.
  - Supporting standards: 7.PR.1, 7.PR.4, 7.PR.5, 7.PR.6, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.FN.6: Compare the growth pattern of exponential to linear or quadratic functions using graphs and tables and recognize how exponential growth exceeds other functions.
  - Supporting standards: 7.PR.1, 7.PR.4, 7.PR.5, 7.PR.6, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9

### Linear Functions, Equations, & Inequalities

### Create & Solve

Students create and solve equations that model linear relationships.

- A1.LFE.1: Represent and solve real-world problems, using linear expressions, equations, and inequalities in one variable.
  - Supporting standards: 6.ALG.3, 6.ALG.4, 6.ALG.7, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 7.NCC.9, 7.PR.6, 8.FN.6, 8.FN.7, 8.ALG.1, 8.ALG.2
- A1.LFE.2: Construct linear functions from arithmetic sequences with and without context.
  - Supporting standards: 7.PR.1, 7.PR.4, 7.PR.6, 8.FN.6, 8.FN.7
- A1.LFE.3: Solve linear formulas for a specified variable.
  - Supporting standards: 6.ALG.3, 6.ALG.4, 6.ALG.7, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 7.NCC.9, 7.PR.6, 8.FN.6, 8.FN.7, 8.ALG.1, 8.ALG.2
- A1.LFE.4: Solve linear equations, linear inequalities, and absolute value equations in one variable, including those with rational number coefficients, and variables on both sides of the equal or inequality sign; solve them fluently, explaining the process used.
  - Supporting standards: 6.NCC.4, 6.ALG.3, 6.ALG.4, 6.ALG.7, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 7.NCC.9, 7.PR.6, 8.FN.6, 8.FN.7, 8.ALG.1, 8.ALG.2

### **Interpret Key Features**

Students interpret key features of equations that model linear relationships.

- A1.LFE.5: Determine the domain and range of linear functions in mathematical problems.
  - Supporting standards: 7.ALG.4, 8.FN.3, 8.FN.8
- A1.LFE.6: Determine reasonable domain and range values of linear functions representing real-world situations, both continuous and discrete; interpret the solution as reasonable or unreasonable in context.
  - Supporting standards: 7.ALG.4, 8.FN.3, 8.FN.8
- A1.LFE.7: Interpret the key features of a linear and absolute value functions that models a relationship between two quantities in a given context.
  - Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.LFE.8: Flexibly use different representations of a linear function, including graphs, tables, and equations.
  - Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.LFE.9: Calculate and interpret the rate of change of a linear function represented in a table, graph, or as an equation in context of real-world and mathematical problems.

- Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.LFE.10: Translate among equivalent forms of equations for linear functions, including standard, point-slope, and slopeintercept forms; recognize that each form reveals key features in a given context.
  - Supporting standards: 6.ALG.4, 7.NCC.9, 7.ALG.1

### Systems of Equations & Inequalities

Students solve systems of equations and inequalities.

- A1.LFE.11: Solve systems of linear equations by substitution, elimination, and graphing with and without a real-world context; understand that the solutions will be the same regardless of the method for solving.
  - Supporting standards: 6.ALG.1, 6.ALG.2, 6.ALG.3, 6.ALG.4, 6.ALG.5, 7.ALG.1, 7.ALG.2, 8.ALG.1, 8.ALG.3
- A1.LFE.12: Solve a system of equations consisting of a linear equation and a quadratic equation in two variables graphically with the assistance of technology.
  - Supporting standards: 6.ALG.1, 6.ALG.2, 6.ALG.3, 6.ALG.4, 6.ALG.5, 7.ALG.1, 7.ALG.2, 8.ALG.1, 8.ALG.3
- A1.LFE.13: Explain why a solution to the equation f(x) = g(x) is the x-coordinate where the y-coordinate of f(x) and g(x) are the same using graphs, tables, or approximations. Include cases where f(x) and/or g(x) are linear, quadratic, absolute value, and exponential.
  - Supporting standards: 6.ALG.1, 6.ALG.2, 6.ALG.3, 6.ALG.4, 6.ALG.5, 7.ALG.1, 7.ALG.2, 8.ALG.1, 8.ALG.3
- A1.LFE.14: Solve linear inequalities and systems of linear inequalities in two variables by graphing.
  - Supporting standards: 6.ALG.1, 6.ALG.2, 6.ALG.3, 6.ALG.4, 6.ALG.5, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 8.ALG.1, 8.ALG.2, 8.ALG.3

### **Graphing & Transformations**

Students graph linear functions, equations, and inequalities.

- A1.LFE.15: Write linear equations that model the relationship between two quantities and produce a graph of the equation.
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 6.ALG.1, 6.ALG.2, 6.ALG.3, 7.ALG.2, 7.ALG.4, 7.PR.6, 8.ALG.1, 8.ALG.2, 8.FN.9
- A1.LFE.16: Graph linear functions expressed as an equation and show intercepts of the graph without technology.
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7
- A1.LFE.17: Graph absolute value functions expressed as an equation with and without technology, showing intercepts and end behavior.
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 6.NCC.4, 7.NCC.3, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7
- A1.LFE.18: Graph and generalize the effect of transformations on linear and absolute value functions.
  - Transformations include: stretches, compressions, vertical, and horizontal
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 6.NCC.4, 7.NCC.3, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7
- **A1.LFE.19:** Given the graph of a linear function, explain the effects of the transformation from the parent function, y = x.
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7

### Statistical Relationships

Students explore linear statistical relationships.

A1.LFE.20: Write linear functions that provide a reasonable fit to data and use them to make predictions, with and without technology; interpret the slope and y-intercept in context.

- Supporting standards: 8.SP.1, 8.SP.2
- A1.LFE.21: Calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association.
  - Supporting standards: 8.SP.1, 8.SP.2
- A1.LFE.22: Compare and contrast correlation and causation in real-world problems.
  - Supporting standards: 8.SP.1, 8.SP.2

### **Quadratic Functions & Equations**

### Create & Solve

Students create and solve equations that model quadratic relationships.

- A1.QFE.1: Represent and solve real-world problems using quadratic expressions and equations in one variable.
  - Supporting standards: 6.ALG.3, 6.ALG.4, 6.ALG.7, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 7.NCC.9, 7.PR.6, 8.NCC.7, 8.NCC.8, 8.FN.6, 8.FN.7, 8.ALG.1, 8.ALG.2
- A1.QFE.2: Write quadratic equations with real number solutions that model the relationship between two quantities and produce a graph of the equation.
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 6.ALG.1, 6.ALG.2, 6.ALG.3, 7.ALG.2, 7.ALG.4, 7.PR.6, 8.ALG.1, 8.ALG.2, 8.FN.9
- A1.QFE.3: Solve quadratic equations with real number solutions, containing one variable, including those with variables on both sides of the equal sign. Equations should be solved by:
  - Graphing,
  - Factoring (including perfect square trinomials and difference of squares binomials),
  - Using the quadratic formula,
  - Completing the square, or
  - Taking the square root.
  - Supporting standards: 6.ALG.1, 6.ALG.2, 6.ALG.3, 6.ALG.4, 6.ALG.5, 7.ALG.1, 7.ALG.2, 7.NCC.9, 8.ALG.1, 8.ALG.2, 8.NCC.7, 8.NCC.8

### **Interpret Key Features**

Students interpret key features of equations that model quadratic relationships.

- A1.QFE.4: Determine the domain and range of quadratic functions in mathematical problems.
  - Supporting standards: 7.ALG.4, 8.FN.3, 8.FN.8
- A1.QFE.5: Determine reasonable domain and range values of quadratic functions representing real-world situations, both continuous and discrete; interpret the solution as reasonable or unreasonable in context.
  - Supporting standards: 7.ALG.4, 8.FN.3, 8.FN.8

- A1.QFE.6: Interpret the key features of a quadratic function that models a relationship between two quantities in a given context.
  - Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.QFE.7: Flexibly use different representations of a quadratic function, including graphs, tables, and equations.
  - Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.QFE.8: Explain how each form of a quadratic expression (standard, factored, and vertex form) identifies different key attributes, using the different forms to interpret quantities in context.
  - Supporting standards: 6.ALG.4, 7.NCC.9, 7.ALG.1, 8.NCC.3, 8.NCC.7, 8.NCC.8, 8.FN.5
- A1.QFE.9: Use factoring and completing the square to create equivalent forms of quadratic functions to reveal key attributes.
  - Supporting standards: 6.ALG.1, 6.ALG.2, 6.ALG.3, 6.ALG.4, 6.ALG.5, 7.ALG.1, 7.ALG.2, 7.NCC.9, 8.ALG.1, 8.ALG.2, 8.NCC.3, 8.NCC.7, 8.NCC.8, 8.FN.5

### **Graphing & Transformations**

Students graph quadratic functions and explore different transformations of  $f(x) = x^2$ .

A1.QFE.10: Graph quadratic functions given as an equation or in function notation, labeling key attributes, without technology.

- Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7
- A1.QFE.11: Graph and describe the effect of transformations on quadratic functions.
  - Transformations include: stretches, compressions, vertical, and horizontal
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7
- A1.QFE.12: Given the graph of a quadratic function, explain the effects of the transformation from the parent function,  $y = x^2$ .
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7

### Statistical Relationships

Students explore quadratic statistical relationships.

- A1.QFE.13: Write quadratic functions that provide a reasonable fit to data and use them to make predictions with technology.
  - Supporting standards: 8.SP.1, 8.SP.2

### **Exponential Functions & Equations**

### Create & Solve

Students create and solve problems that model exponential relationships.

- A1.EFE.1: Represent and solve real-world problems, using exponential equations in one variable.
  - Supporting standards: 6.ALG.3, 6.ALG.4, 6.ALG.7, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 7.NCC.9, 7.PR.6, 8.FN.6, 8.FN.7, 8.ALG.1, 8.ALG.2

- A1.EFE.2: Represent real-world problems (growth, decay, and compound interest), using exponential equations.
  - Supporting standards: 6.ALG.3, 6.ALG.4, 6.ALG.7, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 7.NCC.9, 7.PR.6, 8.FN.6, 8.FN.7, 8.ALG.1, 8.ALG.2
- A1.EFE.3: Construct exponential equations from geometric sequences with and without context.
  - Supporting standards: 6.ALG.3, 6.ALG.4, 6.ALG.7, 6.ALG.8, 7.ALG.1, 7.ALG.2, 7.ALG.3, 7.NCC.9, 7.PR.6, 8.FN.6, 8.FN.7, 8.ALG.1, 8.ALG.2

### Interpret Key Features

Students interpret key features of equations that model exponential relationships.

A1.EFE.4: Determine the domain and range of exponential functions in mathematical problems.

Supporting standards: 7.ALG.4, 8.FN.3, 8.FN.8

- A1.EFE.5: Determine reasonable domain and range values of exponential functions representing real-world situations, both continuous and discrete; interpret the solution as reasonable or unreasonable in context.
  - Supporting standards: 7.ALG.4, 8.FN.3, 8.FN.8
- A1.EFE.6: Interpret the key features of an exponential function that models a relationship between two quantities in a given context.
  - Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.EFE.7: Flexibly use different representations of an exponential function, including graphs, tables, and equations.
  - Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9
- A1.EFE.8: Interpret the quantities in an exponential equation in the context of a real-world problem, including growth, decay, and compound interest.
  - Supporting standards: 5.GM.8, 6.NCC.4, 6.PR.2, 7.PR.1, 7.NCC.3, 7.PR.7, 7.ALG.5, 8.FN.4, 8.FN.5, 8.FN.6, 8.FN.7, 8.FN.8, 8.FN.9

### Graphing

Students graph exponential functions.

- A1.EFE.9: Graph exponential functions that model real-world problems (growth, decay, and compound interest), showing key attributes.
  - Supporting standards: 5.GM.7, 5.GM.8, 6.GM.4, 7.ALG.1, 7.PR.4, 8.FN.1, 8.FN.5, 8.FN.6, 8.FN.7

### Statistical Relationships

Students explore exponential statistical relationships.

- A1.EFE.10: Write exponential functions that provide a reasonable fit to data and use them to make predictions with technology.
  - Supporting standards: 8.SP.1, 8.SP.2

### **Statistics & Probability**

### Numerical Data

Students summarize and describe distributions.

- A1.SP.1: Use box plots and histograms to determine the statistics appropriate to the shape of the data distribution; compare the center and spread of two or more data sets.
  - Supporting standards: 6.SP.2, 6.SP.3, 6.SP.4, 6.SP.5, 6.SP.6, 6.SP.7, 6.SP.8, 7.SP.1, 7.SP.3, 7.SP.4
- A1.SP.2: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points.
  - Supporting standards: 6.SP.4, 6.SP.5, 6.SP.7

### **Bivariate Data**

Students will investigate patterns of association in bivariate data.

- A1.SP.3: Summarize data from two categorical variables in a frequency table; interpret relative frequencies in the context of the data, recognizing data trends and associations.
  - Supporting standards: 6.SP.8, 7.SP.1, 8.SP.3

### Critical Algebra I Supporting Standards:

Fifth Grade

- **5.CAR.12:** Evaluate numerical expressions with parentheses or brackets and exponents with the base of ten, using the Order of Operations.
- **5.CAR.13:** Write simple expressions that record calculations with numbers, interpreting numerical expressions without evaluating them.
- **5.GM.7:** Graph points with whole number coordinates on a coordinate plane in the first quadrant, explaining how the coordinates relate to the horizontal and vertical axes to describe the location of points in the plane.
- **5.GM.8:** Represent real-world and mathematical problems by graphing points in the first quadrant on a coordinate plane, interpreting coordinate values of points in the context of the situation.

### Sixth Grade

- 6.NCC.4: Interpret the absolute value of numbers for positive or negative quantities in a real-world context.
- **6.NCC.11:** Solve real-world and mathematical problems with the greatest common factor of two whole numbers less than or equal to 100.
- 6.PR.2: Calculate unit rates to include unit pricing and constant speed.
- 6.ALG.1: Read and write expressions in real-world or mathematical problems in which letters stand for numbers.
- **6.ALG.2:** Use mathematical terms to identify parts of an expression, including the names of operations, terms, factors, coefficients, variables, and constants.
- **6.ALG.3:** Write and evaluate expressions for given values of variables, using order of operations, including expressions with whole number exponents.

- **6.ALG.4:** Generate equivalent expressions by applying the associative, commutative, distributive, and identity properties.
- **6.ALG.5:** Identify when two expressions are equivalent by using properties of operations including like terms.
- **6.ALG.6:** Use substitution to determine if a given value in a specified set makes an equation or inequality true.
  - Include the following inequality symbols: <, >, ≤, ≥, ≠
- **6.ALG.7:** Write and solve one-step equations in real-world and mathematical problems, involving positive rational numbers and zero.
- **6.ALG.8:** Write, solve, and graph one-step inequalities in real-world and mathematical problems.
- **6.GM.4:** Find and graph pairs of rational numbers in all four quadrants of the coordinate plane in real-world and mathematical problems.
- 6.SP.2: Calculate and interpret any measure of center (mean, median, and mode) of a numerical data set.
- **6.SP.3:** Determine which measure of center (mean or median) is more appropriate to describe the center of data and justify the choice.
- **6.SP.4:** Describe how the mean or median is affected by outliers of a numerical data set.
- **6.SP.5:** Calculate and interpret the measure of variation [range and interquartile range (IQR)] of a numerical data set.
- **6.SP.6:** Determine which measure of variation (range or interquartile range) is more appropriate to describe the shape; justify the choice.
- **6.SP.7:** Represent numerical data on a number line, histogram, and box plot.
- **6.SP.8:** Calculate the relative frequency of an interval of data values when given a histogram.

### Seventh Grade

- **7.NCC.3:** Demonstrate in real-world contexts the distance between two rational numbers on the number line as the absolute value of their differences.
- **7.NCC.7:** Use addition and subtraction with rational numbers in any form to solve multi-step problems in real-world and mathematical contexts.
- **7.NCC.8:** Use multiplication and division with rational numbers in any form to solve multi-step problems in real-world and mathematical contexts.
- **7.NCC.9:** Apply operations with rational numbers involving the order of operations, involving nested grouping symbols.
- **7.PR.1:** Determine the unit rate (constant of proportionality) from tables, graphs, equations, diagrams, or verbal descriptions of proportional relationships.
- **7.PR.4:** Determine whether two quantities represent proportional relationships by using equivalent ratios in a table and by graphing on a coordinate plane.
- **7.PR.5:** Compare two different proportional relationships represented in different forms.
- **7.PR.6:** Create equations in the form of y = mx from tables, verbal descriptions, or graphs.
- **7.PR.7:** Given a graph with a proportional relationship, explain the meaning of a point (x, y) on the graph, including the origin (0, 0) and the unit rate (1, r).
- **7.ALG.1:** Generate and justify equivalent expressions, using properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients within mathematical and real-world problems.
- **7.ALG.2:** Model and solve fluently two-step equations in real-world or mathematical problems.
- **7.ALG.3:** Create, solve, and graph two-step inequalities in real-world and mathematical problems in the forms  $px \pm q > r$ ,  $px \pm q < r$ ,  $px \pm q \geq r$ , and  $px \pm q \leq r$ .
- 7.ALG.4: Write an equation to express two quantities in terms of the dependent and independent variables.
- **7.ALG.5:** Describe the relationship between the dependent and independent variables in an equation using tables and graphs, relating these to the equation.
- **7.SP.1:** Interpret data displayed in a histogram and box plot to answer questions about the data.

- **7.SP.3:** Graph two numerical data sets and compare their variability.
  - Variability includes: range, interquartile range, or mean absolute deviation
- **7.SP.4:** Select an appropriate measure(s) of center or variability and draw valid comparative inferences for two data sets.

### **Eighth Grade**

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8.NCC.3:	Know and apply the properties of integer exponents to generate equivalent numerical expressions.
8.NCC.7:	Solve equations in the form of $x^2 = p$ or $x^3 = p$ where p is a positive rational number.
8.NCC.8:	Evaluate square roots of perfect squares and cube roots of perfect cubes.
8.FN.1:	Graph proportional relationships, interpreting the unit rate as the slope of the graph.
8.FN.3:	Determine whether a relation is a function or not when given a function map, table, graph, equation, or set of ordered pairs.
8.FN.4:	Compare the rate of change (slope) and y-intercept (initial value) of two linear functions each represented in different forms.
	• Functions are represented algebraically, graphically, numerically in tables, or by verbal descriptions.
8.FN.5:	Distinguish between linear and nonlinear functions by comparing graphs and equations.
8.FN.6:	Determine the rate of change (slope) and y-intercept (initial value) from tables, graphs, equations, and verbal
	descriptions of linear relationships.
8.FN.7:	Interpret and explain the meaning of the rate of change (slope) and y-intercept (initial value) of a linear relationship in a real-world context.
8.FN.8:	Analyze a graph by describing the functional relationships between two quantities.
8.FN.9:	Sketch a graph that exhibits qualitative features of a function described verbally.
8.ALG.1:	Analyze and solve one-variable linear equations with rational coefficients containing solutions with one, zero, or infinitely many solutions.
8.ALG.2:	Analyze and solve one-variable linear inequalities with rational coefficients.
8.ALG.3:	Analyze and solve systems of linear equations in the form $y = mx + b$ in real-world or mathematical contexts, graphically and algebraically.
8.SP.1:	Construct scatter plots using bivariate data; determine if the data displays a linear or nonlinear pattern and positive, negative, or no association.
8.SP.2:	Construct straight lines to approximately fit data displaying a linear association when presented in scatter plots.
8.SP.3:	Construct and interpret a relative frequency table, using data from two categorical variables collected from the same subject.

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